Melinda Fidler

Optimizing an Advertising Campaign

Math 1010 Intermediate Algebra Group Project

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Background Information:

Linear Programming is a technique used for optimization of a real-world situation. Examples of optimization include maximizing the number of items that can be manufactured or minimizing the cost of production. The equation that represents the quantity to be optimized is called the objective function, since the objective of the process is to optimize the value. In this project the objective is to maximize the number of people who will be reached by an advertising campaign.

The objective is subject to limitations or constraints that are represented by inequalities. Limitations on the number of items that can be produced, the number of hours that workers are available, and the amount of land a farmer has for crops are examples of constraints that can be represented using inequalities. Broadcasting an infinite number of advertisements is not a realistic goal. In this project one of the constraints will be based on an advertising budget.

Graphing the system of inequalities based on the constraints provides a visual representation of the possible solutions to the problem. If the graph is a closed region, it can be shown that the values that optimize the objective function will occur at one of the "corners" of the region.

The Problem:

In this project your group will solve the following situation:

A local business plans on advertising their new product by purchasing advertisements on the radio and on TV. The business plans to purchase at least 60 total ads and they want to have at least twice as many TV ads as radio ads. Radio ads cost \$20 each and TV ads cost \$80 each. The advertising budget is \$4320. It is estimated that each radio ad will be heard by 2000 listeners and each TV ad will be seen by 1500 people. How many of each type of ad should be purchased to maximize the number of people who will be reached by the advertisements?

Modeling the Problem:

Let X be the number of radio ads that are purchased and Y be the number of TV ads.

1. Write down a linear inequality for the total number of desired ads.

$x + y \geq 60$

2. Write down a linear inequality for the cost of the ads.

$20x + 80y \leq \$4320$

3. Recall that the business wants at least twice as many TV ads as radio ads. Write down a linear inequality that expresses this fact.

x = radio

y = TV

$y \geq 2x$

4. There are two more constraints that must be met. These relate to the fact that there cannot be s negative numbers of advertisements. Write the two inequalities that model these constraints:

$x \geq 0$

$y \geq 0$

5. Next, write down the function for the number of people that will be exposed to the advertisements. This is the Objective Function for the problem.

$P = 2000x + 1500y$

You now have four linear inequalities and an objective function. These together describe the situation. This combined set of inequalities and objective function make up what is known mathematically as a **linear programming** problem. Write all of the inequalities and the objective function together below. This is typically written as a list of constraints, with the objective function last.

$x + y \geq 60$

$20x + 80y \leq 4320$

$y \geq 2x$

$x \geq 0$

$y \geq 0$

$P = 1500y + 2000x$ ($P = 2000x + 1500y$)

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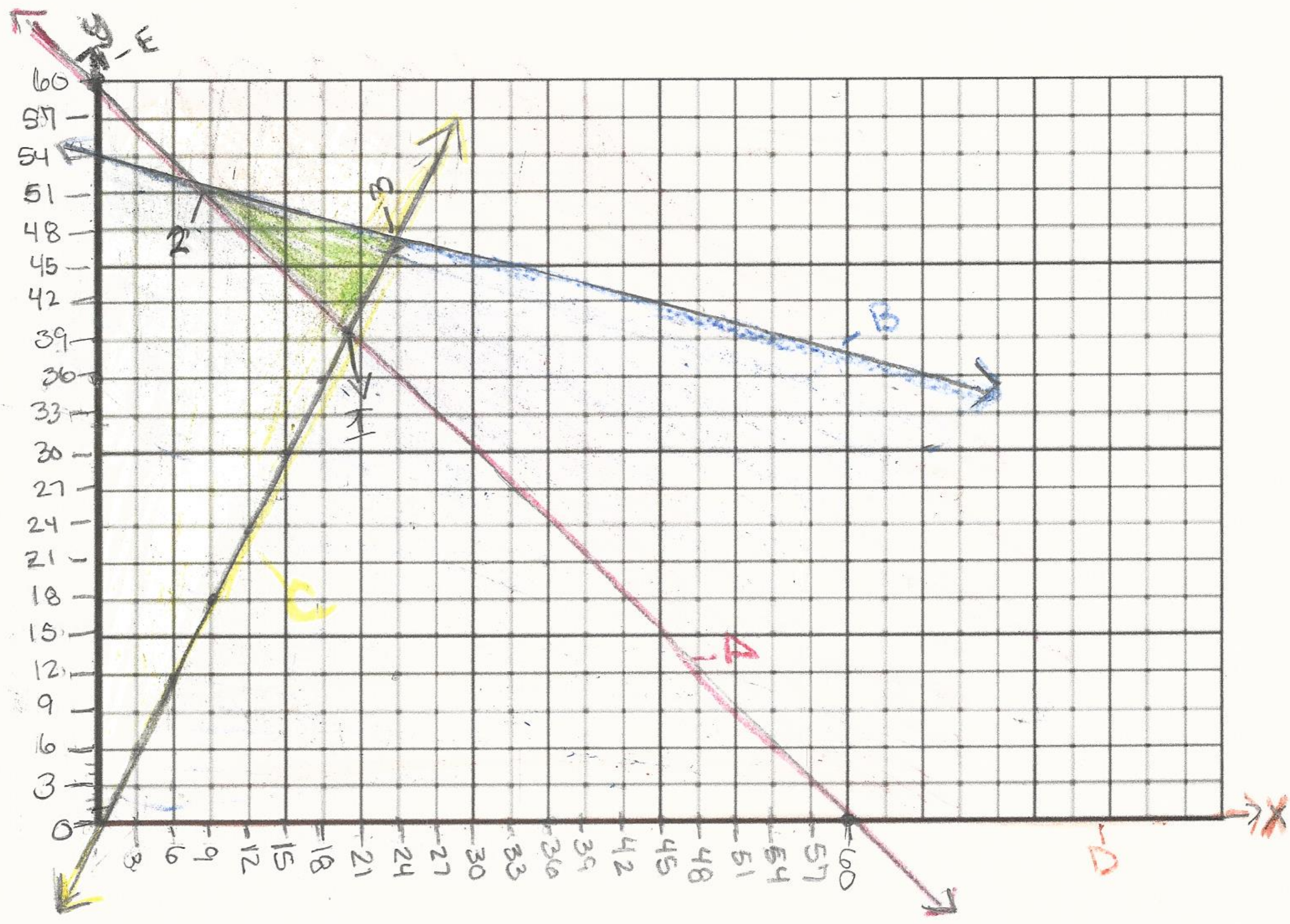
6. To solve this problem, you will need to graph the **intersection** of all five inequalities on one common XY plane. Do this on the grid below. Have the bottom left be the origin, with the horizontal axis representing X and the vertical axis representing Y. Label the axes with what they represent and label your lines as you graph them.



A. $x + y \geq 60 \rightarrow y \geq -x + 60$

B. $y \leq -\frac{1}{4} + 54$

C. $y \geq 2x$

D. $x \geq 0$

E. $y \geq 0$

X can't be negative
y can't be negative

7. The shaded region in the above graph is called the feasible region. Any (x, y) point in the region corresponds to a possible number of radio and TV ads that will meet all the requirements of the problem. However, the values that will maximize the number of people exposed to the ads will occur at one of the vertices or corners of the region. Your region should have three corners. Find the coordinates of these corners by solving the appropriate system of linear equations. Be sure to *show your work* and *label* the (x, y) coordinates of the corners in your graph.

1) $x+y \geq 60$

$y \geq 2x$

$x+(2x) \geq 60$

$\frac{3x}{3} \geq \frac{60}{3}$

$x \geq 20$

$(20)+y \geq 60$
$-20 \quad -20$
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$y \geq 40$

2) $x+y \geq 60$

$20x+80y \leq 4320$

$-20x-20y \leq -1200$
$20x+80y \leq 4320$

$\frac{60y}{60} \leq \frac{3120}{60}$

$y=52$

$y \geq 52$

$x+(52) \geq 60$
$-52 \quad -52$
$x \geq 8$

3) $y \geq 2x$

$20x+80y \leq 4320$

$20x+80(2x) \leq 4320$

$20x+160x \leq 4320$

$\frac{180x}{180} \leq \frac{4320}{180}$

$x \leq 24$

$y \geq 2(24)$

$y \geq 48$

(20, 40) (8, 52) (24, 48)

8. To find which number of radio and TV ads will maximize the number of people who are exposed to the business advertisements, evaluate the objective function P for each of the vertices you found. Show your work.

A (20,40); B (8,52) C (24,48)

$f(P) = 2000x+1500y$

A) $2000(20)+1500(40)$

$40000+60000$

$100{,}000$

B) $2000(8)+1500(52)$

$16000+78000$

$94{,}000$

A. 100,000

B. 94,000

C. 120,000

C) $2000(24)+1500(48)$

$48000+72000$

$120{,}000$

→ 120,000 will maximize the number of people exposed to the ads.

9. Write a sentence describing how many of each type of advertisement should be purchased and what is the maximum number of people who will be exposed to the ad.

The company should purchase 24 radio ads and 48 television ads; this will allow for the maximum number of people, 120,000, to be exposed to the company's new product.

10. Reflective Writing.

Did this project change the way you think about how math can be applied to the real world? Write one paragraph stating what ideas changed and why. If this project did not change the way you think, write how this project gave further evidence to support your existing opinion about applying math. Be specific.

This project did not change my views about the way I think of math in terms of real world application. I have always believed that math is a necessary part of every day life from grocery shopping to the work place. This project is a very good example of how math is applied specifically for a job in any area of buisiness, marketing, television to even, as the math textbook points out on page 266, "battlefield strategy and troop allocation."[1] The project gave me further support for my existing opinion as, while working through problems 6, 7, and 8, I could see these formulas used not only to find how many people would be exposed to an ad, but also how television broadcasting agencies figure out which shows to cancel and which to keep. There is also application in the museum field, as this formula would be good to use to see how many artifacts would be viewed by the maximum number of patrons.

1) Bittinger, Marvin L, Ellenbogen, David J., Johnson, Barbara L. "Intermediate Algebra", Pearson Education Inc. 2014 (pg 266)